Def: An _____ is an _____ of the form

Note: There are two sequences to consider given a series $\sum_{n=0}^{\infty} a_n$.

(a) The sequence of _____

(b) The sequence of _____

Def: If the sequence of _____ converges to a real number S, then the infinite series _____. In this case, S is called the _____, and we write

If the sequence of partial sums diverges, we say that the series _____.

Ex 1 Draw a number line from 0 to 3 below.

(a) On the number line, place a point at 1. Then $1 + \frac{1}{2}$. Then $1 + \frac{1}{2} + \frac{1}{4}$. Then $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

(b) If you were to continue the process indefinitely, to what number would the points that you're drawing keep getting closer? Why?

(c) Write $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^k}$ in sigma notation.

(d) What is $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$?

Ex 2 Suppose r is a constant and consider the series $\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + \cdots$. For which values of r does this series converge?

Def: A _____ is a series whose terms form a

_____:

Note:

Ex 3 Find the exact value of the sum

$$10 - \frac{10}{9} + \frac{10}{81} - \frac{10}{729} + \frac{10}{6561} - \cdots$$

Ex 4 Write the number $5.\overline{279} = 5.279279279\ldots$ as a ratio of two integers.

Ex 5 The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is called the *harmonic series*. Does it converge or diverge?

Thm: (Algebraic Properties of Convergent Series) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be _____. Then the following algebraic properties hold:

(a) The series $\sum_{n=1}^{\infty}(a_n + b_n)$ converges and $\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$.

(b) The series $\sum_{n=1}^{\infty}(a_n - b_n)$ converges and $\sum_{n=1}^{\infty}(a_n - b_n) = \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{\infty} b_n$.

(c) For any real number c, the series $\sum_{n=1}^{\infty} ca_n$ converges and $\sum_{n=1}^{\infty} ca_n = c\sum_{n=1}^{\infty} a_n$.

Ex 6 Does $\sum_{n=1}^{\infty} \left(2^{-\frac{1}{n}} - 2^{-\frac{1}{n+1}}\right)$ converge or diverge? If it converges, to what does it converge?

Q. What are the partial sums of the series $\sum_{n=0}^{\infty} t_n - t_{n+1}$?

Def: Suppose that $(t_n)_{n=0}^{\infty}$ is a convergent sequence with $\lim_{n\to\infty} t_n = L$. Then the series

is called a _____. It converges to

Ex 7 Compute $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

Q. How common are geometric and telescoping series? Will we be able to compute the values of very many sums?